A New Iterative Procedure for Euclidean Projections

Diamond Light Source (DLS) is the UK’s national synchrotron, a particle accelerator that is capable of producing extremely bright light. Beams are generated by accelerating electrons near the speed of light around a ring more than half a kilometre in circumference. They are then directed into laboratories, and used to analyse samples in a variety of research and industry applications, ranging from fossils and solar panels to viruses and vaccines.

In order for this light to be collimated and focused appropriately onto samples, the resulting beam has to be stabilised; this is a large-scale disturbance rejection problem, with 173 outputs, 172 inputs and sampled at frequencies that exceed 10kHz [[1](#References)]. The Oxford Control Group has developed a model predictive control (MPC) system to perform just this task. They also developed several “solvers” (OSQP, Clarabel, Cosmo), that often involve the use of machine learning (ML) algorithms, where a common challenge is often presented by finding Euclidean projections onto convex sets.

To reduce computation time and memory usage in the beam stabilisation algorithm, the group proposed to use the fast gradient method, where the Euclidean projection step is solved using Dykstra’s scheme [[2](#References)]. This is an iterative algorithm for finding the projection onto the intersection of two closed convex subsets in Hilbert space. It is efficient and fast, but can stall under certain conditions [[3](#References)][[4](#References)]. Hence, Dykstra’s method must be modified to avoid stalling, with a focus on polyhedral sets that are encountered in most engineering applications.

The project's principal aim is to develop a new iterative procedure for Euclidean Projections. In particular, it will be the scope of the proposal to answer the following questions:

* Under which conditions does the method not converge for these polyhedral sets?
* Once the conditions for stalling are identified, how could stalling be prevented?

To investigate these problems, we will determine the overall conditions that lead to stalling. Subsequently, we will modify the existing algorithm to prevent stalling using methods from distributed optimisation [[5](#References)]. Finally, we will integrate our procedure into the DLS control system and improve the MATLAB implementation of the MPC system. Next to the significant performance improvements for DLS, the outcomes of this research could also be embedded in solvers. In addition, considering how popular Dykstra’s method is in various ML applications, this project would have a high degree of relevance even outside of the domain of DLS.

References

1. I. Kempf et al., [“Model Predictive Control for Electron Beam Stabilization in a Synchrotron”](https://arxiv.org/abs/2107.01694), arXiv, 2021.
2. I. Kempf et al., [“Fast Gradient Method for Model Predictive Control with Input Rate and Amplitude Constraints”](https://doi.org/10.1016/j.ifacol.2020.12.070), IFAC, 2020.
3. H. H. Bauschke et al., [“On Dykstra’s algorithm: finite convergence, stalling, and the method of alternating projections”](https://arxiv.org/abs/2001.06747v1), arXiv, 2020.
4. C. Perkins, [“A convergence analysis of Dykstra’s algorithm for polyhedral sets”](https://doi.org/10.1137/S0036142900367557), Soc. Ind. Appl. Math. J. Numerical Anal., 2002.
5. L. Romao et al., ["Convergence rate analysis of a subgradient averaging algorithm for distributed optimisation with different constraint sets”](https://ieeexplore.ieee.org/abstract/document/9029252/), IEEE, 2019.